

My Math Path 1–8

Bringing Singapore Math®
to Ontario Classrooms Through
Curriculum-Aligned Resources

REPORT



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What is Singapore Math®?

Singapore Math® incorporates proven educational theories and pedagogical techniques from all over the world—with a strong emphasis on mathematical problem solving and a focus on visualization and critical thinking. Students advance from concrete to pictorial and then to abstract (symbolic) representations to solve problems.

Singapore Math® also promotes positive attitudes toward mathematics—while focusing on student development of skills, concepts, processes, and metacognition. Students are encouraged to reflect on their thinking and learn how to self-regulate so that they can apply these skills to varied problem-solving activities.



Why was Singapore Math® created?

Before the 1980s, Singapore imported its mathematics resources from other countries, and its students were ranked in the lower half of countries in mathematics. Starting in 1980, Singapore began to take a new approach. In order to improve economic growth, the country recognized the need to better educate and train its workforce and to prioritize the development of a strong education system.

To develop its outstanding mathematics program, Singapore studied and incorporated the best practices and research from around the world. This resulted in the creation of a unique approach that uses a purposeful sequence of topics, powerful visual models, and an emphasis on problem solving to build the mastery of mathematical concepts and skills and the positive attitudes and confidence critical for success in the 21st century.

What is Nelson's My Math Path?

Singapore's exceptional mathematics achievement results have attracted worldwide interest in the Singapore approach. Working closely with the Ministry of Education in Singapore, Marshall Cavendish Education began developing customized Singapore Math® programs aligned with local curriculum requirements. In 2009, with Houghton Mifflin Harcourt, they produced the *Math in Focus®* program, aligned with the US Common Core State Standards. In 2015, with Oxford University Press, they published the *Inspire Maths®* program, aligned to the National Curriculum of England. Now, Marshall Cavendish has partnered with Nelson to create *My Math Path*, Grades 1 to 8, the first ever Canadian version of Singapore Math® aligned with Ontario, British Columbia, and Alberta provincial curricula, supported by an extensive teacher professional learning program.

International Singapore Math® achievement results

TIMSS 2015 Results

The Trends in International Mathematics and Science Study (TIMSS) is an assessment administered every four years to collect data on math and science achievement from students in Grades 4 and 8 across 57 countries. It assesses not only what students have learned but their ability to apply their knowledge in new situations and use it to solve problems. Singapore has consistently placed within the top three countries for both Grades 4 and 8 since 1995. Since these incredible results were made public, mathematics educators have been interested in the secret of Singapore's success.

The 2015 TIMSS scores for Grades 4 and 8 are shown in the tables here. Canada ranked 28th and 8th for Grade 4 and Grade 8 mathematics achievement, respectively.

TIMSS 2015 GRADE 4

Singapore	621
Korea, Rep. of	606
Chinese Taipei	599
Hong Kong SAR	594
Japan	586
Russian Federation	538
Kazakhstan	528
Canada	527
Ireland	523
United States	518
England	518

<http://timss2015.org/timss-2015/mathematics/student-achievement/distribution-of-mathematics-achievement/>

TIMSS 2015 GRADE 4

Singapore	618
Hong Kong SAR	615
Korea, Rep. of	608
Chinese Taipei	597
Japan	593
Northern Ireland	570
Russian Federation	564
Norway	549
Ireland	547
England	546
Belgium	546
Kazakhstan	544
Portugal	541
United States	539
Denmark	539
Lithuania	535
Finland	535
Poland	535
Netherlands	530
Hungary	529
Czech Republic	528
Bulgaria	524
Cyprus	523
Germany	522
Slovenia	520
Sweden	519
Serbia	518
Canada	511
Italy	507

<http://timss2015.org/timss-2015/mathematics/student-achievement/distribution-of-mathematics-achievement/>

PISA 2015 Results

The Organisation for Economic Co-operation and Development (OECD) Programme for International Student Assessment (PISA) assesses 15-year-old students in 72 countries in mathematics, science, and reading. These are designed to gauge how well the students master these key subjects in order to be prepared for real-life situations in the adult world. PISA's aim is to provide data that will enable countries to improve their education policies and outcomes. In 2015, PISA focused on science as the major domain, whereas reading and mathematics were the minor domains. Singapore ranked first in all three subjects. The results of the PISA Mathematics Scale 2015 are shown in the table.

Approximately 20 000 15-year-old Canadians from nearly 900 schools participated across the 10 provinces in 2015. Of those students, 4123 were from Ontario. Highlights from Ontario include the following:

- ▶ After a decline between 2003 and 2012, the performance of Ontario students remained stable in 2012 and 2015.

- ▶ Among Canadian provinces, only Quebec had results statistically higher than Ontario's.
- ▶ Ontario students performed just below the Canadian average, and they are still among the top 25% of participating jurisdictions.

Below are the highlights of the 2015 PISA assessment at a provincial level:

- ▶ Students in Quebec, Alberta and British Columbia performed at or above the Canadian average in mathematics.
- ▶ Students in Ontario, Prince Edward Island, Nova Scotia, Newfoundland and Labrador, New Brunswick, Manitoba, and Saskatchewan performed below the Canadian average in mathematics.

PISA MATHEMATICS SCALE 2015

Singapore	564
Hong Kong	548
Macao	544
Chinese Taipei	542
Japan	532
China	531
Korea	524
Switzerland	521
Estonia	520
Canada	516
Netherlands	512

<https://www.oecd.org/pisa/pisa-2015-results-in-focus.pdf>



EQAO 2019 Results

The Education Quality and Accountability Office (EQAO) assessments measure how well Ontario’s public education system is developing students’ reading, writing and mathematics skills. Between 2013 and 2019, the percentage of Grade 3 and Grade 6 students in Ontario who achieved at or above the provincial standard in mathematics declined.

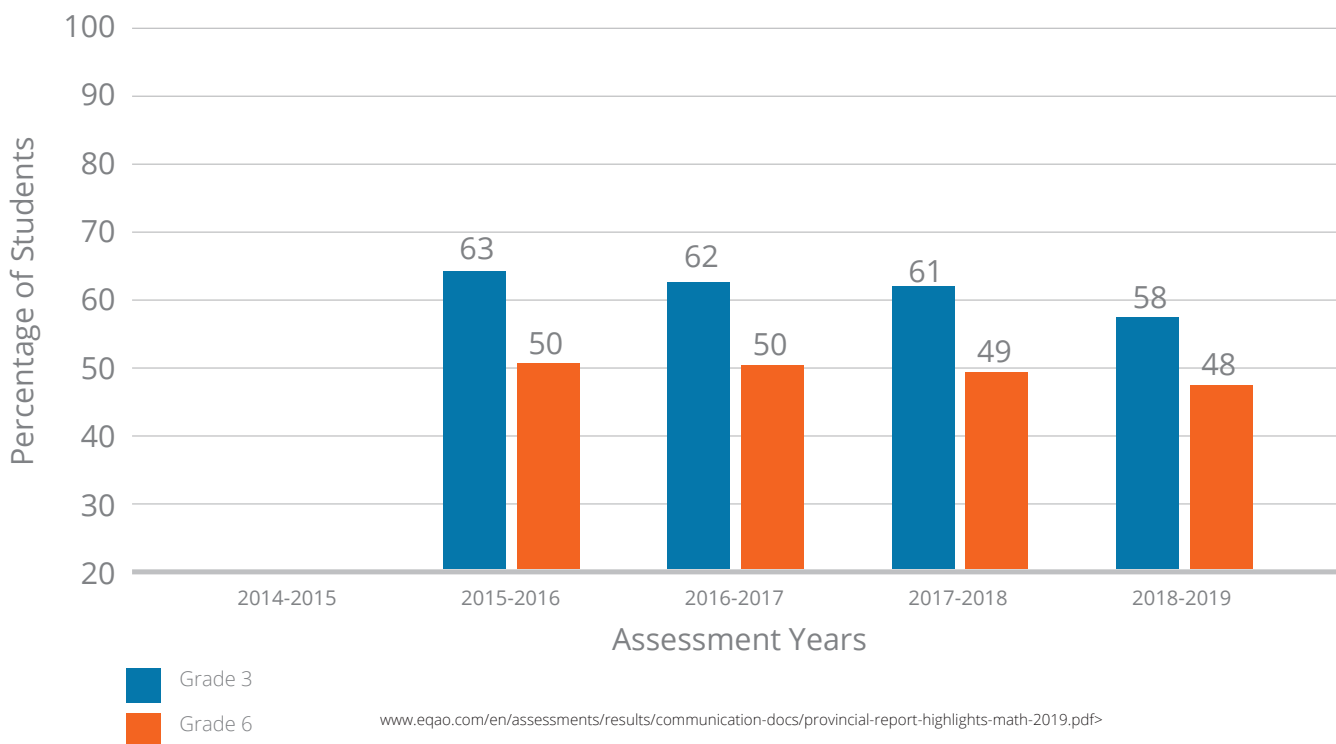
According to the Ministry of Education, Ontario’s students are struggling to meet provincial math standards. The Ontario Renewed Mathematics Strategy that was published in 2016 established a goal of 75 per cent of all elementary students to achieve a level 3 or higher on provincial assessments in mathematics. There has instead been an overall decline in EQAO math scores for the past four years, and this has informed the Ministry’s direction of implementing a greater focus on math fundamentals in Ontario.

The 2019 EQAO results have revealed that on average, of all Grades 3, 6, and 9 students included in the assessment, only 55% like mathematics, and 54% see themselves as good at mathematics. The research also revealed that for Grades 3 and 6, students’ basic knowledge of fundamental mathematics skills is stronger than their ability to apply those skills to a problem or think critically to determine an answer. EQAO poses some key questions to further explore regarding mathematics performance in Ontario:

- ▶ What can be done to promote students’ positive attitudes toward math?
- ▶ In attempts to promote engagement and achievement, how can we further support students’ mathematical problem solving and critical thinking?

www.eqao.com/en/about_eqao/media_room/communication-docs/infographic-math-trends-ontario.pdf

PERCENTAGE OF GRADE 3 AND GRADE 6 STUDENTS AT OR ABOVE THE PROVINCIAL STANDARD (LEVELS 3 AND 4)



Singapore Math® demonstrates proven efficacy in improving mathematics achievement

The efficacy of Singapore Math® in Singapore was demonstrated in the results of TIMSS and PISA, discussed earlier. Efficacy studies have been conducted throughout the United States and the United Kingdom, with students using two programs that follow the Singapore Math® approach: *Math in Focus*® (US) and *Inspire Maths*® (UK). The following studies span urban and rural schools, elementary and middle schools, and economically and culturally diverse student populations.

Overall findings from US *Math in Focus*® efficacy studies

- ▶ 2012–2013, Arkansas, Grades 3, 4, and 5: After one year of use of *Math in Focus*®, at all grade levels examined, the percentage of students scoring proficient or higher was greater than in the previous year; **average gains exceeded 19%**.
- ▶ 2014–2015, Nebraska, Grades 3, 4, and 5: After one year of use of *Math in Focus*®, at all grade levels examined, the percentage of students that met or exceeded the state level of mastery increased over the previous year, with **average gains of over 23%**.
- ▶ 2016–2017, Maine, Wisconsin, New York, Massachusetts, 11 schools, Grades 3, 4, and 5: Students completed a pre-test and a post-test. The *Math in Focus*® program was being used by teachers for the first time. **The average increases in percent correct scores for the total group/low-achieving students/high-achieving students were 16/20/13% for Grade 3, 18/19/12% for Grade 4, and 13/16/12% for Grade 5.**

- ▶ 2018, Connecticut, Idaho, New Jersey, New York, nine schools, Grades 6, 7, and 8: Students completed a pre-test and a post-test. The *Math in Focus*® program was being used by teachers for the first time. **The average increases in correct scores for the total group/low-achieving students/high-achieving students were 24/27/20% for Grade 6, 17/28/8% for Grade 7, and 24/33/17% for Grade 5.**

Overall findings from UK *Inspire Maths*® efficacy study

- ▶ 2017 SATS (national standard attainment tests), Barncroft Primary School, Hampshire: At Key Stage 1, 39% of students were working at greater depth, compared to the national average of 21%. At Key Stage 2, 12% of pupils were working at greater depth, **an increase of 100% on the previous year's results.**



How can Nelson's *My Math Path* transform mathematics education in Ontario?

Builds strong foundations to achieve mastery

Nelson's *My Math Path* is based on the Singapore Math® approach, which focuses on mastery of a limited number of concepts at a time using a purposeful sequence of topics. Presenting each new concept step-by-step and in depth ensures that no key understandings are missed. Closing gaps in a timely fashion ensures that foundational skills learned become permanent parts of a student's math toolkit.

My Math Path emphasizes these fundamental mathematics concepts and skills in every grade. These align with provincial mathematics curricula that focus on learning math fundamentals and developing automaticity that supports critical thinking and problem solving. The sequencing in *My Math Path* ensures students have a strong foundation with Number Sense by working with numbers, understanding number properties, mastering math facts, developing mental math skills, and developing proficiency with operations. Then, as students progress, these number concepts and skills are practised, connected, and applied throughout subsequent topics spanning all the strands of mathematics.

Students need to be fluent with number facts in order to perform mathematical calculations efficiently and accurately, whether mentally or by applying algorithms on paper. The goal is for students to develop automaticity, which is the ability to use skills or perform mathematical procedures with little or no mental effort. Automaticity with math facts also supports students in critical thinking and problem solving.

— Ontario Ministry of Education,
Focusing on the Fundamentals of Math: A Teacher's Guide, 2018, p. 2



Delivers a unique approach to problem solving

Problem solving is at the heart of *My Math Path*.

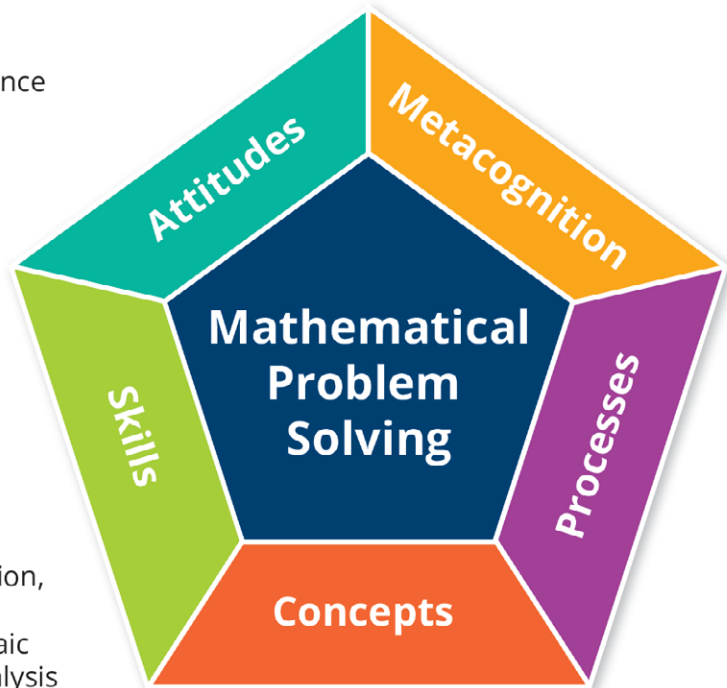
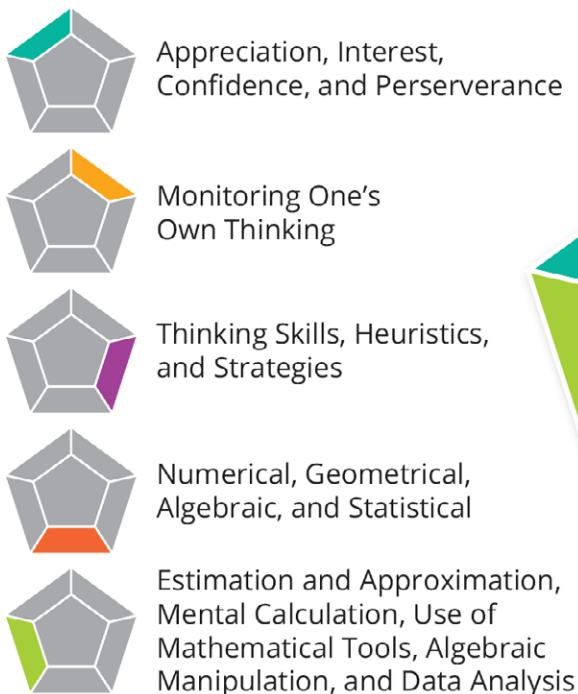
The framework developed by the Singapore Ministry of Education, shown below, emphasizes problem solving and positive attitudes toward mathematics while focusing on student development of skills, concepts, processes, and metacognition.

This framework also enables teachers to convey math concepts progressively from introduction to understanding to fluency, by focusing on the mastery of a limited number of purposefully sequenced concepts at a time.

Each chapter in *My Math Path* contains numerous embedded problem-solving situations so that students learn to flexibly apply their mathematical knowledge. Non-routine, as well as routine problems are included in every grade.

As educators plan student learning experiences, it is important to focus on student understanding and sense making, the interconnectedness of the categories, and the application of skills in problem-solving contexts both in and outside the classroom. The goal should be to provide opportunities for students to come to recognize, informally, how numbers and operations work.

— Ontario Ministry of Education, *Focusing on the Fundamentals of Math*, 2018, p. 4



Each *My Math Path* chapter concludes with Put on Your Thinking Cap!, which challenges students to solve non-routine questions. These problems ask students to draw on deep prior knowledge, as well as recently acquired concepts, combining problem-solving strategies with mathematical process skills. ▶



Students are taught specific problem-solving strategies in a carefully sequenced manner. The most effective of these—model drawing—is used to solve word problems initially, but once acquired as a skill, it becomes useful for solving non-routine problems as well. *My Math Path* teaches students several consistent visual models that they can use to make sense of mathematical relationships and solve problems. They include number bonds, place-value charts, array models, bar models, and more. Each model is fully integrated into the program and carried across grade levels, allowing students to build upon the models and understand increasingly complex concepts.

Grade 1



Put on Your Thinking Cap!

PROBLEM SOLVING
How many beads are there?
Use number bonds to help you.


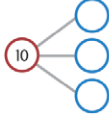
1 There are 6 beads under the two cups.

2 There are 8 beads under the two cups.

3 There are 10 beads under the three cups.

40 Chapter 2 Number Bonds

Lesson 3 **Comparing Two Sets**

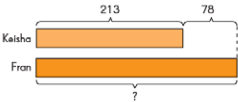
Lesson Objective

- Model addition and subtraction as comparing sets.
- Apply the inverse operations of addition and subtraction.

Vocabulary
compare

LEARN You can use bar models to show **comparing sets to add**.

Keisha has 213 pins in her collection.
 Fran has 78 more pins in her collection.
 How many pins does Fran have in her collection?



$213 + 78 = 291$

Fran has 291 pins in her collection.

Check!
 $291 - 78 = 213$
 $291 - 213 = 78$
 The answer is correct.

Lesson 3 Comparing Two Sets 113

Grade 3

- ◀ Using bar models as a problem-solving tool is introduced in Grade 3. Students become familiar with this systematic way to translate complex word problems into mathematical equations and avoid the common issue of not knowing where to start.

Supports understanding through the Concrete–Pictorial– Abstract framework

My Math Path emphasizes conceptual understanding, which is achieved through the Concrete–Pictorial– Abstract (CPA) framework. The most significant theory to inform this framework is Jerome Bruner’s theory on the representations of mathematical concepts according to different levels of children’s thinking. Children at a certain age, in general, can only conceptualize mathematical concepts depending on their level of mental development. Many recent studies have shown the positive impact of the CPA framework on student success in mathematics. A list of further readings can be found at the end of this report.

Research shows that children cannot depend too much on concrete representation, as they need to move on to the next level so that they can conceptualize abstract (complicated) situations using pictorial representation,

such as in the model approach used in *My Math Path*. Although not all challenging problems can be tackled using the model method, it plays a significant role in helping at-level and below-level students solve problems based on their level of thinking at the concrete and semi-concrete operational stage.

At the concrete stage, students use hands-on manipulatives to explore new concepts. From there, they move to the pictorial stage to see these ideas represented by diagrams and discover how models help show the relationships between numbers. The consistent and deliberate use of visual models across grade levels allows students to apply mathematics competently and confidently to both routine and novel mathematical situations. Finally, students connect their concrete experiences and pictorial representations to abstract symbols.

Concrete

Pictorial

Abstract

Concrete
Manipulatives are used to explain abstract mathematical concepts.

Pictorial
Pictures, models, and diagrams are used to present examples with solutions.

Grade 1

LEARN Number bonds can help you add.

How many toy buses are there in all?

part 3 whole 5
part 2 whole 5

$3 + 2 = 5$
 $2 + 3 = 5$

There are 5 toy buses in all.
You can add in any order.
 $3 + 2 = 2 + 3$

Guided Learning

Add. Use number bonds to help you.

7 How many paper clips are there in all?

part 4 whole 7
part 3 whole 7

part 3 whole 7
part 4 whole 7

$4 + 3 = 7$
 $3 + 4 = 7$

There are 7 paper clips in all.

$4 + 3 = 7$
 $3 + 4 = 7$

52 Chapter 3 Addition Facts to 10

Promotes positive attitudes toward mathematics

In Singapore, efforts to develop positive attitudes and to improve metacognitive skills are evident in all aspects of mathematics learning, including classroom learning materials and the information sent home to students and families. Through professional learning, teachers develop a greater understanding of the methodologies used to encourage positive attitudes towards problem solving using the Concrete–Pictorial–Abstract framework.

The 2015 PISA assessment revealed that both provincially and Canada-wide, students with high levels of mathematics confidence performed the equivalent of two proficiency levels higher on the combined mathematics scale than did students with low levels of mathematics confidence. Students with high levels of mathematics anxiety performed the equivalent of one proficiency level lower in mathematics than did students with low levels of mathematics anxiety.

Students' attitudes have a significant effect on how they approach problem solving and how well they succeed in mathematics. [...] Students need to understand that, for some mathematics problems, there may be several ways to arrive at the correct answer. They also need to believe that they are capable of finding solutions.

— Ontario Ministry of Education, The Ontario Curriculum Grades 1–8, Mathematics, 2005, p. 26

Efficacy studies on the adaptations of Singapore Math®, including *Inspire Maths*® by Oxford University Press, and *Math in Focus*® by Houghton Mifflin Harcourt, revealed exceptional improvements in students' mathematics proficiency but have also shown increases in student motivation and engagement. One notable instance of this is a research study conducted on *Inspire Maths*®:

- ▶ 2015–2016, 12 schools in the UK, 576 Year 1 pupils: The *Inspire Maths*® program was evaluated for one school year using a randomized control trial. The research combined child assessments with classroom observations and interviews with teachers. The study found significant gains in progress after two terms' use of the program. Teachers reported that the program boosted students' motivation and engagement and could be used creatively and flexibly. It both benefited students' math achievement and supported teachers' professional development.

The unique problem-solving approach in *My Math Path* shows students, through non-routine problems, that there is more than one way to solve the problem at hand. By giving students a variety of tools that support every learner, from model drawing, to scaffolding through the Concrete–Pictorial–Abstract approach, students' mathematical proficiency will improve. In turn, they should grow in confidence in their ability to problem solve and reason mathematically.

Builds teacher capacity and confidence

Not only does *My Math Path* promote positive attitudes and proficiency in math for students, it also gives teachers the support and tools they need to confidently teach mathematics. There is evidence that many primary teachers lack confidence in their own mathematical ability. Most educators who encounter Singapore Math® for the first time have a “light-bulb moment” whereby they finally “get the math.” Furthermore, they go on to say how they wish they had been taught how to solve problems using strategies inherent to the Singapore Math® approach, such as bar modeling, when they were students.

My Math Path offers a variety of professional learning opportunities and tools—print, online, and in-person—to support teachers so they can effectively use and teach the strategies and framework integral to the Singapore Math® approach. It's no surprise that this increase in teacher efficacy will lead to a higher degree of student achievement and success.

Prepares students for their best future

A strong mathematical foundation and confidence in one's abilities is important for success in high-school math and can dramatically broaden students' career pathways. *My Math Path* sets students up for success by giving them a set of essential skills for daily living, employment, and responsible citizenship in the future.

Nelson's *My Math Path* brings proven efficacy to the Ontario classroom

The ultimate objective of *My Math Path* is to develop a program that enables students to empower their thinking and develop skills that will help prepare them for their future. The extensive research behind Singapore Math® has produced resources across the world that helps students with mathematical concepts, skills, problem solving, and mathematical investigations. Nelson's *My Math Path* brings this exceptional resource to Canadian classrooms as the first-ever provincially aligned version.

Making sure that students have a strong understanding of the fundamentals of math is one of the best ways to prepare them for success, now and in the future. What students learn today will help best position them to solve everyday problems and to increase their employability in tomorrow's economy.

— Ontario Ministry of Education, A Parent's Guide to the Fundamentals of Math, Grades 1-8, (2018), p. 1

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